Physics: Introduction to Problem Solving

Scuola Normale Superiore, Pisa, Italy

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SCUOLA NORMALE SUPERIORE

Introduction

We are students and PhD candidates at Scuola Normale Superiore (Pisa, Italy) who took part in the Physics Olympiad back in high school days. These slides were used between June and July 2024 for the orientation courses organized by SNS, aimed at students in their penultimate year of high school. Approximately 260 students took part in our classes, the majority of them enjoyed their time! Slides are organized as follows:

1. Introductions
2. Problems
   - Logic and Common Sense
   - Dimensional Analysis
   - Estimations
   - Classical Mechanics
   - Miscellanea

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- Graduated from SNS and Univesity of Pisa, Condensed Matter Physics;
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- Graduate in Theoretical Physics, SNS;
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- Creator of the SNS Physics School;
- National coordinator of the Physics Team Competition;
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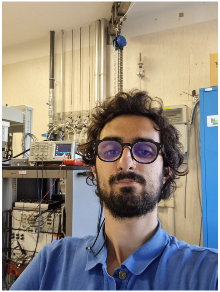
- Nanoscience student at SNS;
- Soon starting a PhD at TU Delft (Netherlands);
- Organizer of the Team Physics Competition and the SNS Physics School;
- Ranked second at PLANCKS 2023.



Daniele Romano Pavarini

- Graduated in Theoretical Physics, SNS;
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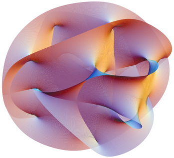


Problems

Lead or Straw?

Which weighs more: $1\,\mathrm{kg}$ *of lead or* $1\,\mathrm{kg}$ *of straw?*

Which weighs more: $1\,\text{kg}$ of lead or $1\,\text{kg}$ of straw?

Definition of mass: **the amount of protons, neutrons, and electrons contained in a body.**

Mathematical translation: $m = N_\mathrm{p} m_\mathrm{p} + N_\mathrm{n} m_\mathrm{n} + N_\mathrm{e} m_\mathrm{e}$.

The masses are the same, but the scale measures a force, not a mass. Apart from the scale's reactive force (called the *normal* force), what other forces are involved?

- Weight force $F_p = mg$,
- Archimedes' force $F_A = \rho_\mathrm{air}\, gV$.

On the scale, we read a quantity that is proportional to the *normal* force:

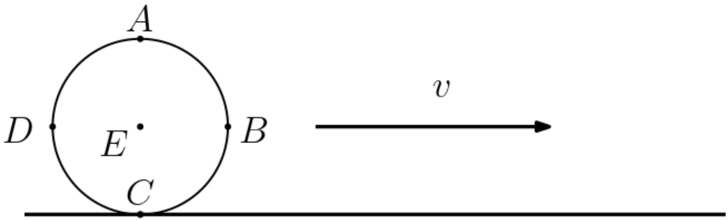
$$m^* = \frac{F_p - F_A}{g} = m - \rho_{air}\, V.$$

Straw is much less dense than lead, so the volume it occupies is much larger. Consequently, the buoyant force acting on the straw is larger than that acting on the lead. Therefore, the mass read on the scale is larger for lead!

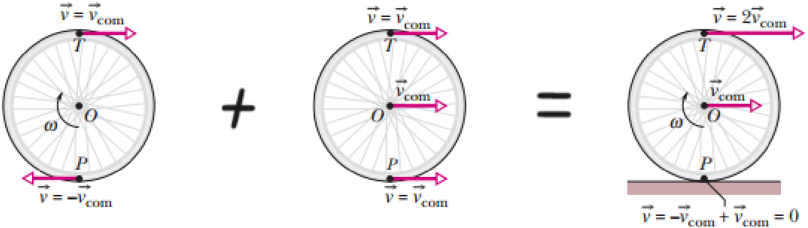
Rolling Wheel

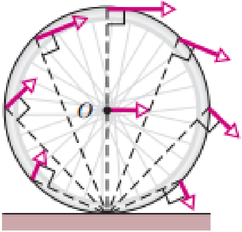
SNS Admission exam, 2013 (simplified persion)

Consider a wheel rolling without slipping on a horizontal plane. If we took a picture of it, which point would appear the clearest?



Rolling Wheel: Composition of Velocities



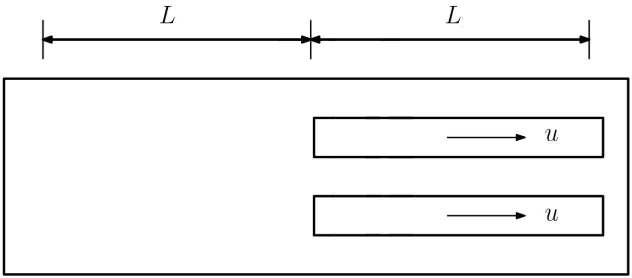


Rolling Wheel: the lowest point is still and clear!

Conveyor Belt

SNS Admission Exam, 2012 (simplified version)

The corridor that separates the airport waiting area from your flight gate is half floor and half long conveyor belts. Normally you move by walking, but if you are late, you can also run. Since you are not able to run the entire length of the corridor, should you run on the conveyor belts or on the floor[1]*?*



[1] Later, we will assume you can run only on one of the two sections at a time.

Conveyor Belt

Quantities involved:

- $L = 150\,\text{m}$, length of the two sections;
- $u = 1.5\,\text{m/s}$, speed of the conveyor belt;
- $v_{\text{walk}} \approx u$, walking speed;
- $v_{\text{run}} = 6\,\text{m/s}$, running speed;
- $t_{\text{run}} = 15\,\text{s}$, time we can run before getting tired;

- Running on floor:

$$t_1 = t_{\text{run}} + \frac{L - v_{\text{run}}\, t_{\text{run}}}{v_{\text{walk}}} + \frac{L}{v_{\text{walk}} + u}.$$

- Running on conveyor belt:

$$t_2 = \frac{L}{v_{\text{walk}}} + t_{\text{run}} + \frac{L - (v_{\text{run}} + u)\, t_{\text{run}}}{v_{\text{walk}} + u}.$$

Conveyor Belt

The expressions on the previous slide are true only under the assumption that $t_{\text{run}} < L/(v_{\text{run}} + u)$. The proposed numerical values satisfy this inequality. Let's calculate the difference between the two times:

$$t_1 - t_2 = -\frac{v_{\text{run}}\, t_{\text{run}}}{v_{\text{walk}}} + \frac{(v_{\text{run}} + u)\, t_{\text{run}}}{v_{\text{walk}} + u} = \\ = \frac{u(v_{\text{walk}} - v_{\text{run}})\, t_{\text{run}}}{(v_{\text{walk}} + u)\, v_{\text{walk}}} < 0.$$

Thus, running on the floor saves time!

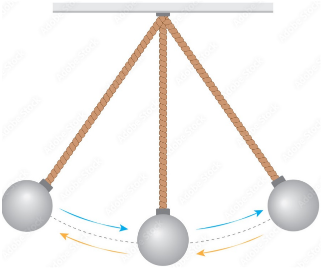
Note that the time difference is independent of the length of the corridor! This characteristic is true only when the two sections are of equal length.

Substituting the proposed values from the previous slide, the time saved is $9\,$s.

Dimensional Analysis: The Period of a Pendulum

Consider a simple pendulum, which is an object of mass m *suspended from the end of an inextensible string of length* ℓ *in Earth's gravitational field, where the acceleration is* g*. The other end of the string is attached to the ceiling.*

How is its period related to the other quantities that characterize the system?



Dimensional Analysis: The Period of a Pendulum

The quantities that might contribute to the result are:

- the mass m of the object, with dimensions $[m] = \mathsf{kg}$;
- the magnitude of the acceleration due to gravity g, with dimensions $[g] = \mathsf{m/s^2}$;
- the length ℓ of the string, with dimensions $[\ell] = \mathsf{m}$.

We look for a combination of these quantities of the form

$$T = C\, m^\alpha g^\beta \ell^\gamma,$$

where C is a dimensionless constant and the exponents need to be determined. The units of this equation are

$$\mathsf{s} = \mathsf{kg}^\alpha \left(\frac{\mathsf{m}}{\mathsf{s}^2}\right)^\beta \mathsf{m}^\gamma.$$

To ensure the equation has consistent dimensions on both sides, we need

$$1 = -2\beta, \quad 0 = \alpha, \quad 0 = \beta + \gamma.$$

Dimensional Analysis: The Period of a Pendulum

The solution to the system is $\alpha = 0$, $\beta = -1/2$, $\gamma = 1/2$. The answer is therefore

$$T = C\, m^0\, g^{-1/2}\, \ell^{1/2} = C\sqrt{\frac{\ell}{g}}.$$

Indeed, we know that, for small oscillations, the exact result is

$$T = 2\pi\sqrt{\frac{\ell}{g}}.$$

For large oscillations, the dimensionless constant is different, but the dependence on ℓ and g is identical and cannot be otherwise.

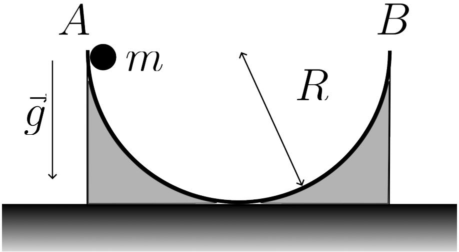
Result: we guessed the dependence on the other quantities forgetting about forces and motion!

Scaling the Time

GaS Training Round, 2023

An object of mass $m = 1\,\mathrm{kg}$ *moves frictionlessly on a semicircular track fixed to the ground. We know that the object takes* $t_0 = 2.2\,\mathrm{s}$ *to travel from* A *to* B *starting from rest.*

If the radius of the track is doubled, how much time will it take for the object to travel from A *to* B*?*



Scaling the Time

The quantities that might contribute to the result are:

- the mass m of the object, with dimensions $[m] = \mathsf{kg}$;
- the acceleration due to gravity g, with dimensions $[g] = \mathsf{m}/\mathsf{s}^2$;
- the radius R of the track, with dimensions $[R] = \mathsf{m}$.

We look for a combination of these quantities of the form

$$T = C\, m^{\alpha} g^{\beta} R^{\gamma},$$

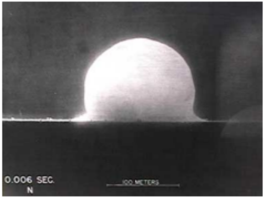
where C is a dimensionless constant and the exponents need to be determined. The units of this equation are

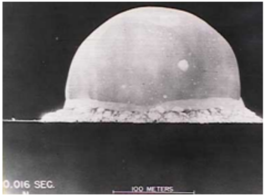
$$\mathsf{s} = \mathsf{kg}^{\alpha} \left(\frac{\mathsf{m}}{\mathsf{s}^2}\right)^{\beta} \mathsf{m}^{\gamma},$$

which yields $\alpha = 0$, $\beta = -1/2$, $\gamma = 1/2$. The required time is therefore proportional to the square root of the radius of the track. Thus,

$$t_0 = C\sqrt{\frac{R_0}{g}}, \qquad t' = C\sqrt{\frac{R'}{g}} \quad \Longrightarrow \quad t' = t_0\sqrt{\frac{R'}{R_0}} = \sqrt{2}\, t_0.$$

When an atomic bomb explodes, a fireball forms and rapidly expands. The fireball from the first atomic bomb had a radius of $80\,\mathrm{m}$ *after* $0.006\,\mathrm{s}$ *from the explosion. Knowing that the way the fireball expands over time depends only on the energy released by the bomb and the air density, how large was the radius of the fireball after* $0.016\,\mathrm{s}$ *from the explosion?*





Atomic Fireball

The text itself tells us that the radius of the fireball depends on the energy E released by the bomb and the air density ρ. What other quantity is missing? To have an expanding front over time, we need time t itself!

Let's find a combination of t, E, and ρ with dimensions of length, of the form

$$R = C\, t^{\alpha} E^{\beta} \rho^{\gamma},$$

where α, β, and γ are real numbers. Converting this expression into dimensional equations, we get

$$[\mathsf{m}] = [\mathsf{s}]^{\alpha} \left(\frac{[\mathsf{kg}]\,[\mathsf{m}]^2}{[\mathsf{s}]^2} \right)^{\beta} \left(\frac{[\mathsf{kg}]}{[\mathsf{m}]^3} \right)^{\gamma}.$$

When using dimensional analysis, converting quantities to units of measurement must be done using only fundamental units! Thus, we need to convert any J, W, N... into expressions containing m, s, kg, K...

Atomic Fireball

Equating the dimensions on both sides, we get the system

$$\begin{cases} 1 = 2\beta - 3\gamma, \\ 0 = \alpha - 2\beta, \\ 0 = \beta + \gamma, \end{cases}$$

which has the solution $\alpha = 2/5$, $\beta = 1/5$, $\gamma = -1/5$. Therefore, we can write

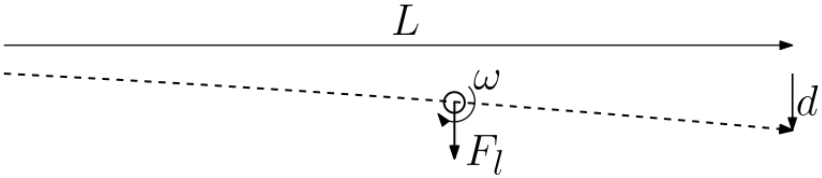
$$R(t) = C\,t^{2/5}E^{1/5}\rho^{-1/5},$$

where C is a dimensionless constant, and thus

$$R(t_2) = R(t_1)\left(\frac{t_2}{t_1}\right)^{2/5} \approx 106\,\mathrm{m},$$

where $t_1 = 0.006\,\mathrm{s}$ and $t_2 = 0.016\,\mathrm{s}$.

A baseball can be thrown with a spin around its axis to achieve a "curveball".



Given the following quantities, estimate the lateral force F_l and the deflection d using dimensional analysis. Assume that the force F_l is directly proportional to ω and that the dimensionless constant C is of the order of unity.

$$m = 0.145\,\text{kg}, \qquad R = 3.7\,\text{cm}, \qquad v = 36\,\text{m/s},$$
$$\omega = 227\,\text{rad/s}, \qquad L = 18\,\text{m}, \qquad \rho_a = 1.2\,\text{kg/m}^3.$$

Baseball

The force F_l is certainly independent of the mass of the ball - it is a force due to the fluid in which the body moves - and the distance traveled. Using the assumption that F_l is directly proportional to ω, we have

$$F_l = C \rho_a^{\alpha} \, \omega \, v^{\beta} R^{\gamma},$$

where the units are

$$\frac{\mathsf{kg} \cdot \mathsf{m}}{\mathsf{s}^2} = \left(\frac{\mathsf{kg}}{\mathsf{m}^3} \right)^{\alpha} \left(\frac{1}{\mathsf{s}} \right) \left(\frac{\mathsf{m}}{\mathsf{s}} \right)^{\beta} \mathsf{m}^{\gamma},$$

from which $\alpha = 1$, $\beta = 1$, and $\gamma = 3$. In conclusion, given that the text suggests $C \approx 1$, we find

$$F_l \approx \rho_a \, \omega \, v R^3 \approx 0.5 \, \mathsf{N}.$$

Baseball

Given that a small deflection is expected, it can be assumed that the lateral force is always perpendicular to the initial direction of flight. The ball follows a parabolic trajectory and the deflection is given by

$$d \approx \frac{1}{2} a_l \, t^2,$$

where

$$a_l = \frac{F_l}{m} \qquad \text{and} \qquad t = \frac{L}{v}.$$

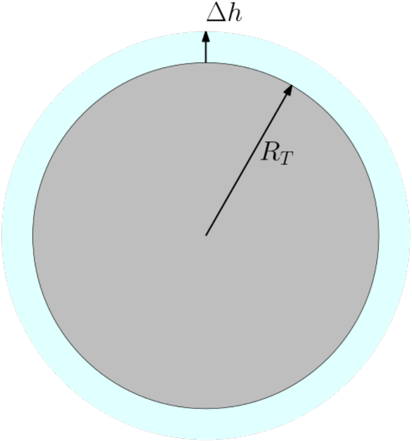
The final result is

$$d \approx \frac{F_l L^2}{2mv^2} = \frac{\rho_a \, \omega L^2 R^3}{2mv} = 0.44 \, \text{m}.$$

Question: Can a similar result be found using **only** dimensional analysis? No! There would be too many unknowns and too few equations.

How many molecules are there in the Earth's atmosphere?

GaS Qualification Round, 2023



How many molecules are there in the Earth's atmosphere?

1. The Earth's gravity attracts molecules in the atmosphere: it is the weight of all the molecules that causes atmospheric pressure P_0.
2. Most of the molecules are located within a few tens of kilometers from the Earth's surface, so we can consider the gravitational acceleration g to be uniform for each molecule.

The weight of all the molecules is $M_{tot}\, g$. Thus, the pressure at the Earth's surface is

$$P_0 = \frac{M_{tot}\, g}{S} = \frac{M_{tot} g}{4\pi R_T^2} = \frac{N_{tot} \langle m \rangle g}{4\pi R_T^2}.$$

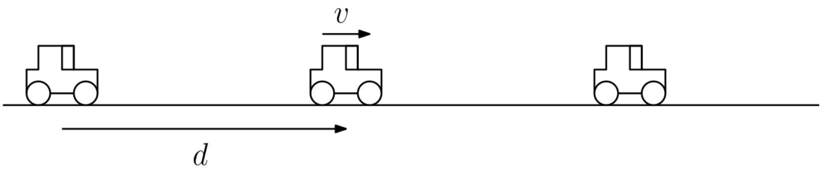
The total mass is the product of the number of molecules in the air and the average mass of a single molecule. More than 97% of the molecules in the air are nitrogen (N_2, 28 nucleons) and oxygen (O_2, 32 nucleons), so the average number of nucleons per molecule is about 30. Hence,

$$N_{tot} = \frac{4\pi R_T^2 P_0}{30\, m_n g} \approx 10^{44}.$$

Road Flow Rate

SNS Admission Exam, 2022 (slightly modified problem)

What is the maximum flow rate of a single-lane road (in number of vehicles per hour), if all cars travel at $70\,\mathrm{km/h}$ *and follow the appropriate safety conditions?*



The flow rate of a road is the number of cars passing a specific point per unit of time. For simplicity, we consider all cars to be the same and traveling at the same cruising speed v. How many cars are present on a stretch of road of length L?

$$n(v) = \frac{L}{d(v)},$$

where d is the distance between the centers of two consecutive cars.

Road Flow Rate

Consideration: Safety conditions imply that the density of cars on a given stretch of road decreases as their speed increases. The distance d is a function of speed! The flow rate, i.e., the number of cars passing a certain point per unit of time, is

$$p(v) = \frac{\Delta N}{\Delta t} = \frac{\rho(v)\, v\, \Delta t}{\Delta t} = \frac{n(v)}{L} v = \frac{v}{d(v)},$$

where $\rho(v)$ is the number of cars per unit length of the road. What is $d(v)$?

$$\begin{aligned} d(v) &= l_{\text{car}} + d_{\text{safety}} = \\ &= l_{\text{car}} + d_{\text{reaction}} + d_{\text{braking}} = \\ &= l_{\text{car}} + v t_{\text{reaction}} + \frac{v^2}{2\mu g}, \end{aligned}$$

where μ is the coefficient of friction between the tires and the road. After braking, the motion is uniformly decelerated.

Road Flow rate

Thus, the flow rate is

$$p(v) = \frac{v}{d(v)} = \frac{v}{l_{\text{car}} + v\, t_{\text{reaction}} + \frac{v^2}{2\mu g}}.$$

Let's use some numbers:

$$\begin{aligned} l_{\text{car}} &\approx 4\,\text{m}, \\ t_{\text{reaction}} &\approx 0.3\,\text{s}, \\ \mu &\approx 0.8, \\ v &= 20\,\text{m/s}. \end{aligned}$$

Substituting, we get

$$\begin{aligned} p(v = 20\,\text{m/s}) &\approx 0.57\,\text{car/s} \\ &\approx 2000\,\text{cars/h}. \end{aligned}$$

Glass of Water in the Ocean

GaS Final Round, 2023

Martin the fisherman leans out from his boat, fills a glass with seawater (with a molar mass of $18\,\mathrm{g\,mol^{-1}}$*), and then empties it back into the sea immediately. Many years later, when the water taken in the glass has completely mixed into all the world's oceans (which cover about* 70% *of the Earth's surface and have an average depth of* $4\,\mathrm{km}$*), Martin is back on his boat. He fills the same glass a second time.*

How many molecules did Martin catch both times with the glass?

Glass of Water in the Ocean

We can set up the following proportion:

$$x : n_{\text{glass}} = n_{\text{glass}} : N_{\text{ocean}}, \quad \Longrightarrow \quad x = \frac{n_{\text{glass}}^2}{N_{\text{ocean}}}.$$

Let's calculate the volume of the oceans: it is a thin layer with an average thickness $h = 4\,\text{km}$ on a sphere of radius R_T, so its volume is

$$V_{\text{ocean}} = f\, 4\pi R_T^2 h,$$

where $f = 0.7$. Let M_{a} be the molecular mass of water and N_A the Avogadro number, then the number of water molecules in the ocean is

$$N_{\text{ocean}} = V_{\text{ocean}}\, N_A\, \rho_{\text{a}}/M_{\text{a}},$$

while those in the glass are

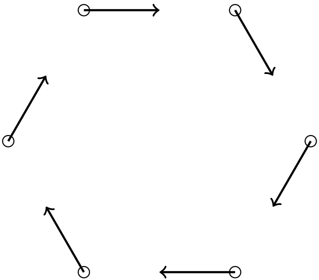
$$n_{\text{glass}} = V_{\text{glass}}\, N_A\, \rho_{\text{a}}/M_{\text{a}}.$$

Thus,

$$x = \frac{n_{\text{glass}}^2}{N_{\text{ocean}}} = \frac{V_{\text{glass}}^2\, \rho_{\text{a}} N_A}{4\pi R_T^2 h f M_a} \approx 10^3.$$

Attractive Ants

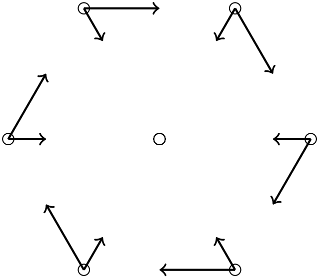
SNS Admission Exam, 2011 (slightly modified problem)



Six ants are initially placed at the vertices of a hexagon with side $l = 1\,\mathrm{m}$*. At a certain moment, they all start moving with a constant speed* $v = 1\,\mathrm{m/s}$*, each pointing instantaneously towards the position of the nearest ant, in the clockwise direction.*

How long does it take for the six ants to meet at the center of the original hexagon?

Attractive Ants

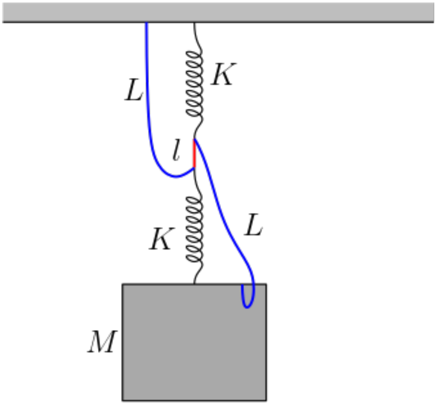


By symmetry, the ants will remain arranged in a hexagon until the end! This lets us realize that the only component of velocity that affects the result is the radial one, which is constant in time and equal to

$$v_r = v\cos(60^\circ) = 0.5\,\mathrm{m/s}.$$

The radial distance that ants must travel to meet at the center is equal to l, so the time required is $t = 2\,\mathrm{s}$.

Springs in Series and Parallel

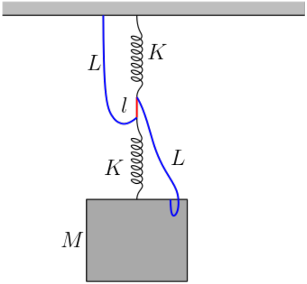


What happens if we cut the red rope?

Springs in Series

Usually, the block stabilizes at a lower height, but in general, it depends on the parameters. For example, consider

$$M = 1\,\mathrm{kg}, \quad K = 250\,\mathrm{N/m}, \quad l = 1\,\mathrm{cm}, \quad L = 6\,\mathrm{cm}, \quad g = 10\,\mathrm{m/s^2}.$$



For two identical springs in series, it holds that

$$K_{\text{series}} = \frac{K}{2},$$

from which it follows

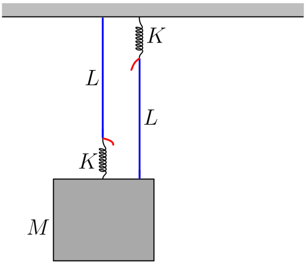
$$L_i = \frac{2Mg}{K} + l = 8\,\mathrm{cm} + 1\,\mathrm{cm} = 9\,\mathrm{cm}.$$

For two different springs in series, it holds that

$$K_{\text{series}} = \left[\frac{1}{K_1} + \frac{1}{K_2}\right]^{-1}.$$

Springs in Parallel

$$M = 1\,\text{kg}, \quad K = 250\,\text{N/m}, \quad l = 1\,\text{cm}, \quad L = 6\,\text{cm}, \quad g = 10\,\text{m/s}^2.$$



For two identical springs in parallel, it holds that

$$K_{\text{parallel}} = 2K,$$

from which it follows

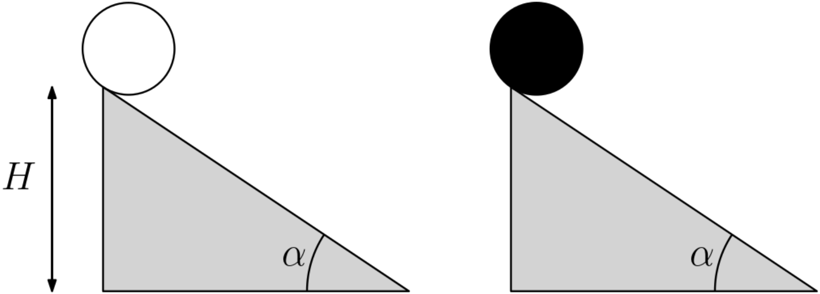
$$L_f = \frac{Mg}{2K} + L = 2\,\text{cm} + 6\,\text{cm} = 8\,\text{cm},$$

$$L_f = 8\,\text{cm} < 9\,\text{cm} = L_i.$$

So, the block stabilizes at a height greater than the initial one!

For two different springs in parallel, it holds that $K_{\text{parallel}} = K_1 + K_2$.

Empty Cylinder or Solid Cylinder?



If they start rolling at the same instant, which one reaches the bottom first?

Empty Cylinder or Solid Cylinder?

The condition for pure rolling is $\omega = v/R$. The energy conservation implies

$$mgH = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left[mv^2 + I\frac{v^2}{R^2}\right].$$

The final velocity of the cylinder is

$$v = \sqrt{\frac{2mgH}{m + I/R^2}}, \quad \text{with} \quad I_{\text{empty}} = mR^2, \qquad I_{\text{solid}} = \frac{1}{2}mR^2,$$

which leads to

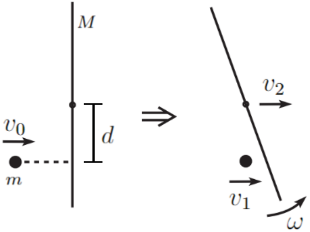
$$v_{\text{solid}} > v_{\text{empty}} \quad \Longrightarrow \quad T_{\text{solid}} < T_{\text{empty}}.$$

Try to calculate the exact results:

$$T_{\text{empty}} = \sqrt{\frac{4H}{g\sin^2\alpha}}, \qquad T_{\text{solid}} = \sqrt{\frac{3H}{g\sin^2\alpha}}.$$

How can it be inferred that they must be independent of mass?

Collision with a Rod



1. *In the case of elastic collision, what are the velocities of the two bodies after the collision? What is the angular velocity of the rod around its center of mass?*
2. *In the case of a completely inelastic collision, what are the velocities of the two bodies after the collision? What is the angular velocity of the object rod + ball around its center of mass?*

Collision with a Rod: Elastic Case

In this case, momentum, kinetic energy, and angular momentum are all conserved.

We choose the point of contact as the pivot for calculating angular momentum because this way, the contribution of the ball to the total angular momentum is zero both before and after the collision.

$$\begin{cases} mv_0 = m{\color{red}v_1} + M{\color{red}v_2}, \\ \frac{1}{2}mv_0^2 = \frac{1}{2}m{\color{red}v_1}^2 + \frac{1}{2}M{\color{red}v_2}^2 + \frac{1}{2}I{\color{red}\omega}^2, \\ 0 = I{\color{red}\omega} - Md{\color{red}v_2}, \end{cases}$$

where $I = \frac{1}{12}ml^2$ is the moment of inertia of the rod calculated with respect to its center of mass. We have a system of three equations and three unknowns. The solution is

$$v_1 = v_0 \frac{1 - \frac{M}{m} + \frac{Md^2}{I}}{1 + \frac{M}{m} + \frac{Md^2}{I}}, \quad v_2 = \frac{2v_0}{1 + \frac{M}{m} + \frac{Md^2}{I}}, \quad \omega = \frac{2v_0 Md}{I\left(1 + \frac{M}{m} + \frac{Md^2}{I}\right)}.$$

Collision with a Rod: Inelastic Case

For inelastic collisions, kinetic energy is not conserved. The condition of *perfect inelasticity* means that the two bodies stick together after the collision.

$$\begin{cases} mv_0 = (m+M)\,v_{\mathrm{cm}}, \\ 0 = I_{\mathrm{cm}}\omega - (m+M)\left(d - \frac{m}{m+M}d\right)v_{\mathrm{cm}} = I_{\mathrm{cm}}\omega - Md\,v_{\mathrm{cm}}, \end{cases}$$

where I_{cm} is the moment of inertia of the new object, calculated with respect to its center of mass. We have a system of two equations and two unknowns, whose solution is

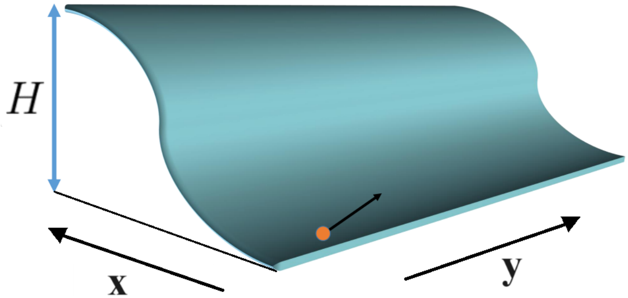
$$\begin{cases} v_{\mathrm{cm}} = \frac{m}{m+M}v_0, \\ \omega = \frac{Mm\,d}{(m+M)I_{\mathrm{cm}}}v_0. \end{cases}$$

Try to obtain the result

$$I_{\mathrm{cm}} = \frac{1}{12}Ml^2 + \frac{Mm}{M+m}d^2.$$

Climbing a Smooth Step

SNS Admission 1967



Two horizontal semi-planes, separated by a vertical distance H, are connected by a smooth step as shown in the figure. A ball, which can move frictionlessly on these surfaces without detaching from them, is launched with a speed of magnitude v on the lower semi-plane.

What is the maximum ratio $\frac{v_y}{v_x}$ for which the ball can overcome the step?

Climbing a Smooth Step

- The ball is subjected to two forces: the weight force and the reaction force from the smooth step.
- In this case, the reaction force is always perpendicular to the ball's velocity, so it **does not do work**. This allows us to use energy conservation:

$$\frac{1}{2}mv^2 = \frac{1}{2}m\left(v_x^2 + v_y^2\right) = \frac{1}{2}m\left(v_{fx}^2 + v_{fy}^2\right) + mgH,$$

  where v_{fx} and v_{fy} are the velocity components once the ball has overcome the step.
- Given the shape of the step, there is no force applied along the y direction, so

$$v_y = v_{fy}.$$

Climbing a Smooth Step

- Combining the two previous results, we get

$$\frac{1}{2}mv_x^2 = \frac{1}{2}mv_{fx}^2 + mgH.$$

- The minimum value of v_x that allows overcoming the step is the one corresponding to the condition $v_{fx} = 0$, so

$$v_x = \sqrt{2gH}.$$

Consequently,

$$\frac{v_y}{v_x} = \sqrt{\frac{v^2}{2gH} - 1}.$$

You can try to prove that, in the case of overcoming the step, the initial and final velocity vectors obey a sort of Snell's law!

Opening a Refrigerator
SNS Admission Exam, 2019

On a hot summer day, you decide to get a cold drink from the fridge. You open the door, take what you want, and close it. Shortly after, you get hungry, so you reopen the fridge to get a snack, but you notice that the door shows much more resistance.

How can this phenomenon be qualitatively explained? How much force is needed to open the refrigerator door?

Opening a Refrigerator

Qualitative Explanation: Refrigerators have valves that ensure that the air inside has a pressure almost equal to atmospheric pressure. However, the process of molecules exchange is not instantaneous.

- **The refrigerator has been closed for a long time**: the equilibrium process had enough time to take place, so the pressure inside is almost equal to atmospheric pressure.
- **The refrigerator was closed recently**: the air inside had time to cool down, but the pressure didn't have enough time to equilibrate. Therefore, more force is needed to open the door.

Let's estimate the force required to open the fridge in the second case. From the ideal gas law, we get

$$\frac{p_0 V}{T_0} = \frac{p_1 V}{T_1} \implies p_1 = p_0 \frac{T_1}{T_0} \approx p_0 \frac{275\,\mathrm{K}}{300\,\mathrm{K}} \approx 0.9\, p_0.$$

In order to open the fridge, we require a force which is greater than the one due to the pressure difference.

Opening a Refrigerator

The force is

$$F = S\Delta p \approx S\,(p_0 - p_1) \approx 0.1\,p_0\,S,$$

where

$$p_0 \approx 10^5\,\mathsf{Pa},$$
$$S \approx 0.5\,\mathsf{m}^2.$$

Thus

$$F \approx 0.5 \times 10^4\,\mathsf{N} \approx 500\,\mathsf{kg} \cdot g.$$

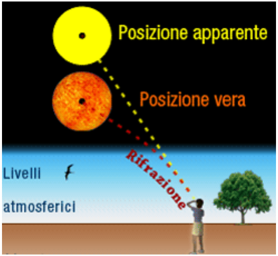
This number is very high! What's wrong? Actually, it's not the force that needs to balance the pressure force, but since the door has hinges on one side, the moments of the forces must also balance (this is a second-class lever). This makes the force required about half of the value found above, but it still huge. Evidently, the fridge is not airtight! Additionally, we assumed that all the air that just entered immediately cooled down to 275 K. This is not realistic, especially for a fairly empty fridge. Experience tells us that the force is that needed to lift an object of a few kilograms.

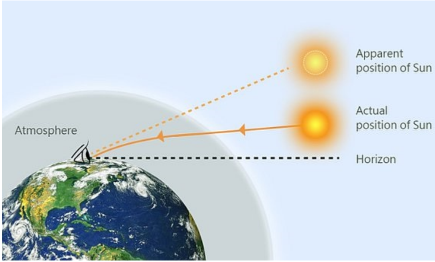
Apparent Position of Stars

SNS Admission Exam, 2014

The refractive index of the atmosphere continuously increases from the higher atmosphere, where $n = 1$, *to the Earth's surface, where* $n = 1.0003$.

Find a relation between the angle θ_{true} *and the angle* θ_{apparent}, *where* θ_{true} *is the angle that defines the true direction of light coming from a star, while* θ_{apparent} *defines the apparent direction of the star as perceived by an observer on the Earth's surface. Ignore Earth's curvature.*

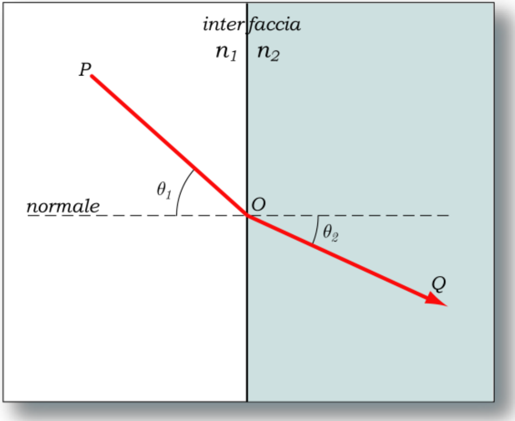




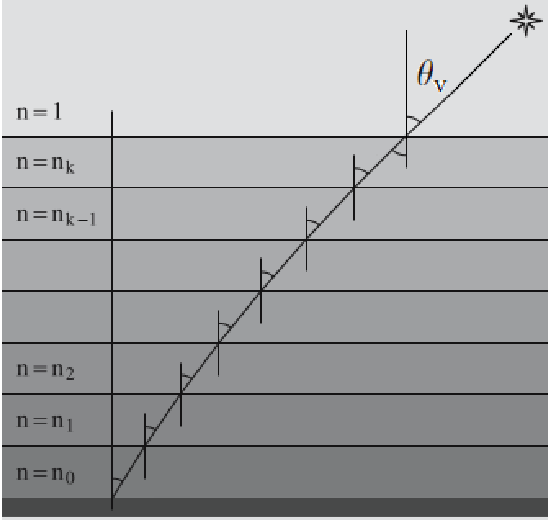
Apparent Position of Stars

Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$



Apparent Position of Stars



Apparent Position of Stars

Let us divide the atmosphere in an infinite amount of infinitesimal layers. If the layers have infinitesimal thickness, the refractive index is homogeneous within each layer. This allows us to consider the light ray as straight within a single layer. Therefore, Snell's Law applies to each pair of consecutive layers:

$$
\begin{aligned}
&n_0 \sin\theta_0 = n_1 \sin\theta_1,\\
&n_1 \sin\theta_1 = n_2 \sin\theta_2,\\
&\dots\dots\dots\dots\dots\dots\dots\dots\\
&n_{k-1} \sin\theta_{k-1} = n_k \sin\theta_k,\\
&n_k \sin\theta_k = n_{\text{vacuum}} \sin\theta_{\text{vacuum}}.
\end{aligned}
$$

The chain of equalities implies that

$$n_0 \sin\theta_0 = n_{\text{vacuum}} \sin\theta_{\text{vacuum}} \Longrightarrow 1.0003 \cdot \sin\theta_{\text{apparent}} = 1 \cdot \sin\theta_{\text{true}}$$

$$\Longrightarrow \theta_{\text{apparent}} = \arcsin\left(\frac{\sin\theta_{\text{true}}}{1.0003}\right) < \theta_{\text{true}}.$$

Slides by:

- Daniele Battesimo Provenzano,
- Nico Kleijne,
- Fabio Zoratti,
- Antonio Lombardi.

This https://uz.sns.it/~Batpov/slides_eng.pdf is the link to the PDF file of this lecture.

At this link https://uz.sns.it/~Batpov/Tecniche_di_Problem_Solving_in_Fisica.pdf you will find a handout containing some of the topics covered. Feel free to spread it around!

DBP: *“Dedicato ai miei compagni d’avventura, Fisici Bestiali”.*


That's all Folks!